UNIT-05

Monday

15/09/2025 **STANDARD NORMAL & SAMPLING DISTRIBUTION**

Total area under the curve equals 1.

$Z = \frac{x-\mu}{\sigma}$ is called standard normal variate

Imp

Ex: If a student's height is converted into Z-score to compare with others, the resulting values follows a standard normal distribution.

Definition:

A continuous random variable Z with the interval $(-\infty, \infty)$ has the probability density function $\phi(Z) = \frac{1}{\sqrt{2\pi}} e^{-Z^2/2}, -\infty < Z < \infty$ is called standard normal distribution. It is usually denoted by $Z \sim N(0,1)$ Z-distribution

It is used to find probabilities and critical values in statistical analysis

*Derivation of p.d.f of standard normal distribution:

$Z = \frac{x-\mu}{\sigma}$ by using one variable Jacobian transformation method.

$Z = \frac{x-\mu}{\sigma}$ by using one

$\Rightarrow x = \mu + \sigma Z$

$dx = \sigma dZ$

$J = \frac{dx}{dZ} = \sigma$

$|J| = \sigma \quad (\because \sigma > 0)$

limits If $x \to \infty$, $Z \to \infty$

If $x \to \mp\infty$, $z \to \mp\infty$

$$\phi_z(z) = f_x(x)\,|J|$$

$$= f_x(\mu + \sigma z) \cdot \sigma$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{\mu + \sigma z - \mu}{\sigma}\right)^2} \sigma$$

$$\boxed{\phi_z(z) = \frac{1}{\sqrt{2\pi}} \cancel{\sigma}\, e^{-\frac{z^2}{2}}}$$

Note: The important feature of standard normal distribution is mean "zero" and variance is "one"

i.e If $x \sim N(\mu, \sigma^2)$ then $z \sim N(0,1)$

* standard normal integral:

$$\int_{-\infty}^{\infty} \phi(z)\,dz = 1 \Rightarrow \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz = 1$$

(or) $$\int_{-\infty}^{\infty} e^{-z^2/2}\,dz = \sqrt{2\pi}$$

* M.G.F of standard normal distribution:

P.d.f of standard normal distribution is

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty < z < \infty.$$

M.G.F of z is, $M_z(t) = E(e^{tz})$

$$= \int e^{tz} \phi(z)\,dz$$

$$= \int_{-\infty}^{\infty} e^{tz} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2(z^2 - 2tz)}\,dz$$

$$= \frac{1}{\sqrt{2\pi}} \int e^{-1/2 (z^2 - 2tz + t^2 - t^2)} dz$$

$$= \frac{e^{t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2 (z-t)^2} dz$$

let $z - t = \mu \Rightarrow dz = d\mu$

limits : $-\infty < \mu < \infty$

$$= \frac{e^{t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{\mu^2}{2}} d\mu = \frac{e^{t^2/2}}{\sqrt{2\pi}} \sqrt{2\pi}$$

$$\boxed{M_z(t) = e^{t^2/2}}$$

The M.G.F of standard normal distribution is $e^{t^2/2}$

* Mean and variance of standard normal distribution through M.G.F :

The M.G.F of standard normal distribution is $M_z(t) = e^{t^2/2}$

$$= 1 + \frac{t^2}{2} + \frac{1}{2!}\left(\frac{t^2}{2}\right)^2 + \frac{1}{3!}\left(\frac{t^2}{3}\right)^3 \ldots$$

$$M_x(t) = 1 + \frac{t^2}{2} + \frac{t^4}{8} + \frac{t^6}{48} + \ldots$$

Mean $= \mu_1' =$ Coefficient of $\frac{t}{1!}$ in $M_x(t) = 0$

$$\text{Variance} = \mu_2 = \mu_2' - \mu'^2$$
$$= 1 - 0$$
$$= 1$$

Variance of standard normal distribution is '1'.

Imp

17/09/2025 Wednesday

* Student - t distribution:

It was introduced by W.S. GOSSET.

A continuous random variable t with probability density function:

$$f(t) = \frac{1}{\sqrt{n-1}\, B\left(\frac{1}{2}, \frac{n-1}{2}\right)} \frac{1}{\left(1+\frac{t^2}{n-1}\right)^{\frac{(n-1)+1}{2}}}; \quad -\infty < t < \infty.$$

is called Student t-distribution.

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* Applications of Student t-distribution:

1. It is used to test the significant difference between the sample mean ($\bar{x}$) and the population mean (μ).
2. It is used to test significance difference between two sample means.
3. It is used to test the significance of

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an observed sample correlation coefficient and sample regression coefficient.

4. It is used to test the significance of observed partial and multiple correlation coefficients.

* Definition of F- distribution:

F- distribution was introduced by Snetecor. A continuous random variable F with probability density function

$$f(F) = \frac{\left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}}}{B\left(\frac{n_1}{2}, \frac{n_2}{2}\right)} \cdot \frac{F^{\frac{n_1}{2}-1}}{\left(1+\frac{n_1}{n_2}F\right)^{\frac{n_1+n_2}{2}}} ; \quad 0 < F < \infty$$

is called F- distribution with (n_1, n_2) degree of freedom.

* Applications of F- distribution:

1. It is used to test the equality of population variance.

2. It is used to test the equality of several means.

3. It is used to test the significance of linearity of regression.

4. It is used to test the significance of an observed multiple correlation coefficient.

5. To test the significance of an observed sample correlation ratio.

Imp

* Definition of (chi) χ^2 distribution:

(chi) χ^2 is a continuous random variable with probability density function

$$f(\chi^2) = \frac{1}{2^{n/2} \cdot \Gamma(n/2)} e^{-\chi^2/2} (\chi^2)^{\frac{n}{2}-1} ; 0 < \chi^2 < \infty$$

is called a χ^2 distribution with n degrees of freedom.

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* Applications of χ^2 distribution:

1. It is used to test goodness of fit.

2. It is used to test independence of attributes.

3. It is used to test the significant difference between the sampl...

and population variance.

4. To test the homogenity of independent of estimates of population variance

5. To test the homogenity of the independent estimates of the population correlation coefficient.

19/09/2025 Friday

* properties of t- distribution:

1. t- statistic range from $-\infty$ to ∞
2. Mean of t- distribution is zero.
3. Variance of t- distribution is $n/(n-2)$
4. The probability curve of t- distribution is symmetrical about $t=0$
5. Kurtosis of t- distribution is lypto kurtic.
6. For large values of degree of freedom (n), t- distribution tends to normal distribution.

9. Moment Generating function of t-distribution does not exists.

8. T- distribution is assymptotic to x-axis

* **properties of F-distribution:**

1. limits of F-statistic ranges from 0 to ∞

2. Mean of F-distribution is $\frac{n_2}{n_2-1}$

3. Variance of F-distribution is $\frac{2n_2^2(n_1+n_2-2)}{n_1(n_2-2)^2(n_2-4)}$

4. Mode of F-distribution is $\frac{(n_1-n_2)}{n_1(n_2+2)}$ and mode of F-distribution is always less than unity

5. F-distribution is positive skewed

6. F-distribution curve asympotic to the right tail

7. The reciprocal of f-statistic is also a F-statistic

V.Imp
* **properties of χ^2-distribution:-**

1. ~~the~~ χ^2 variate lies in between 0 to ∞.

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2. mean of χ^2 distribution (n)

3. Variance of χ^2 distribution is (2n).

Standard deviation of χ^2 distribution is $\sqrt{2n}$.

4. Moment generating function of χ^2 distribution $(1-2t)^{-n/2}$

5. characteristic function of χ^2 distribution is $(1-2it)^{-n/2}$

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